The Initial Teacher Education Research Project

Report on mathematics courses for Intermediate Phase student teachers at five universities

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List of Acronyms

BEd	Bachelor of Education
C	Concepts
CCK	Common content knowledge
CKT-M	Content knowledge for teaching mathematics
FET	Further education and training phase (Grades 10 to 12 – Senior High)
FP	Foundation phase (Grades R to 3 – Junior Primary)
HEI	Higher Education Institution
IP	Intermediate phase (Grades 4 to 6 – Senior Primary)
K	Knowledge
M	Mathematics
Ma	Mathematics content
Me	Mathematics methodology
ML	Mathematical literacy
MP	Mathematical practices
MRTEQ	Minimum Requirements for Teacher Education Qualifications
NSC	National Senior Certificate
P	Practice
PGCE	Postgraduate Certificate in Education
PUFM	Profound understanding of fundamental mathematics
RP	Routine procedures
SCK	Specialised content knowledge
SenPrim	Senior Primary School
SP	Senior phase (Grades 7[1] to 9 – Junior High)
TEDS-M	Teacher Education and Development Study in Mathematics
TIMSS	Third International Mathematics and Science Study

[1] Grade 7 is usually included in the primary school and forms part of the senior primary school, although the curriculum places it in the senior phase together with Grades 8 and 9 which make up the junior high school.

1 Introduction

An outline of the Initial Teacher Education Research Project (ITERP), including the research methods employed to derive the data discussed below, are contained in Taylor 2014. ITERP is investigating the nature and quality of initial teacher education programmes offered by universities and the extent to which these programmes are meeting the needs of the South African schooling system. The four components of the programme are:

1. The content of teacher education programmes for students training as Intermediate Phase (IP) teachers at five universities, together with the instruments used to assess the practice teaching undertaken by these students. The present report is one of four describing the findings of this component.

2. Case studies of newly qualified teachers (NQTs) in their first two years of teaching.

3. Survey of all final year (BEd and PGCE) students in 2013, tracking them into the workplace for two years.

4. Recommendations for ITE in the IP; action arising from the findings and recommendations.

Further details are provided in Deacon 2012; Reed 2014; Rusznyak and Bertram 2014; Deacon 2014; and Taylor 2014. These will all be available at www.jet.org.za.

The courses investigated in this report were those in existence in 2013 (or in the years leading up to 2013). Some of the institutions indicated that they were in the process of revising their courses in light of the Minimum Requirements for Teacher Education Qualifications (MRTEQ) Policy. As these revisions had not been implemented at the time of data collection, they are not discussed in any depth in this report.

The report begins by providing basic information about the weighting of the mathematics courses in the BEd programme for IP teachers, the entrance requirements and the number of students on the courses. A number of factors influence the kind of courses the institutions are able to offer. Clearly class size, student background and time available have a significant impact on the nature of the courses an institution is able to offer. In addition, each of the institutions discussed has been uniquely affected by mergers with and incorporation into the universities that commenced in 2003. These contextual factors are documented in the detailed reports on each of the institutions and the discussion of the mathematics offerings that follow below can be better understood in conjunction with these detailed reports.

This report then goes on to discuss the scope and depth of the mathematics courses. There is no agreement, across campuses, on the curriculum for mathematics courses for prospective IP teachers, whether they intend to specialise as maths teachers or not. There is also considerable difficulty in judging the depth of a mathematics course. The nature of mathematics as a strong vertical discourse in which new knowledge is built on prior knowledge means that it is reasonable to assume that later knowledge (e.g. calculus) is “deeper” than the fundamentals (e.g. arithmetic and algebra) on which it rests. However it is also possible to tackle basic mathematics (e.g. work with number and arithmetic) at a very sophisticated, conceptual level and to tackle advanced mathematics (e.g. calculus) at an entirely procedural level.

The debate about the nature of mathematics appropriate for teachers (and particularly IP teachers) is far from settled. Research does not support the notion of "the more higher mathematics courses the better" for teachers (see for example, Monk, 1994 and Begle, 1972). In relation to mathematics courses for prospective primary school teachers, the appropriateness of higher level mathematics courses (e.g. courses on calculus, linear algebra and other mathematics courses typically studied in a Science degree at university) has been questioned.

Considerable argument has been put forward for the necessity of a particular kind of understanding for mathematics teachers. Ma (1999), in her comparison of Chinese and American primary school teachers, argues for the importance of what she terms a "profound understanding of fundamental mathematics (PUFM)" and proposes that teachers need a deep, connected and flexible understanding of the mathematical content that they are expected to teach. Hill et al (2005) argue that what they refer to as content knowledge for teaching mathematics (CKT-M) is a particular kind of mathematical knowledge that needs to inform the work that teachers do in classrooms. This would include, amongst other skills, the ability to use different representations to illustrate a mathematical concept, the ability to analyse the appropriateness of alternative methods to approach a mathematical problem and the ability to analyse the origin of misconceptions. The research of Hill and her colleagues has indicated a link between teachers' scores on a test for CKT-M and the quality of their teaching. However, more work needs to be done to understand what kinds of pre-service mathematics courses best deliver good CKT-M or PUFM. The complexities around describing the depth of the mathematics courses meant that the analysis of the courses' content was multifaceted. This is described in detail in section 4.1.

The nature of mathematics methodology courses (i.e. courses intended to guide prospective teachers in how to teach mathematics) is also a contested area. There is considerable variation in terms of the actual content to be covered in such courses. In addition, there is variation in terms of the way in which that content can be approached. There is clearly a strong link to the practice of teaching (i.e. what the teacher will actually do in the classroom). However, the field of mathematics education has a research base and the location of teacher education in universities allows for the possibility of an approach more strongly located in the discipline of mathematics education. The methodology courses were thus analysed in relation to these dimensions. This is also elaborated in section 4.1.

This report deals with the mathematics courses offered to prospective IP teachers. For ease of reference this group of students will be referred to as **IP students** throughout this report. Two sub-groups will be discussed: the first is the group of IP students who are specialising in mathematics and these will be referred to as **IP Maths students;** the second group consists of IP students who are not specialising in mathematics and these will be referred to as **IP non-Maths students** in what follows.

2 Basic information

Table 1 and Table 2 summarise the mathematics courses for prospective IP teachers at the five case study institutions. Table 1 does so for those who will specialise in the mathematics and Table 2 for those who will not. The tables give information on:

a) Number of students: Where information was available this is given for each of the four years of the B.Ed.; alternatively approximate numbers of the cohort are given. In order to give an indication of the proportion of IP students at each institution who specialize in mathematics the percentage of IP students taking mathematics is given in brackets in the student numbers column in Table 1.

b) Entrance requirements: In Table 1 any additional entrance requirements for students specialising in mathematics are noted. These are indicated as the minimum percentages required for NSC Mathematics (M) or Mathematical Literacy (ML), apart from Institution A which requires students to achieve 65% for a compulsory maths test at the end of their first year.

c) Maths credits: The total number of credits in the mathematics and mathematics methodology courses is shown. The number of mathematics and mathematics methodology credits is also given as a percentage of the total number of credits in the BEd in order to give an indication of the weighting of these courses in the degree.

e) Additional support offered: This notes whether there was any particular additional support is provided to assist students who are struggling with mathematics. This is support beyond that provided during office hours of lecturers' office hours or by well-designed lectures.

2.1 Basic information about the mathematics offering for prospective mathematics specialists in the intermediate phase

Table 1: Basic information about the mathematics offerings for prospective IP mathematics specialists

HEI	Approx. number of IP maths specialists per year (prop of all IP students)	Entrance Requirements	Maths Credits	Percentage of total credits	Additional support offered
A	36 (10%)	65% for 1st year compulsory maths test	100	21%	
B	65 (36%)	M 50% ML not allowed	128	25%[2]	Tutorials
C	100-200 (±10%)[3]	Pass (30%) in M or ML	108	23%	One optional tutorial per semester for some courses
D	101 (24%)	M 40% or 50%	120	24%	
E	Very variable From 58 (52%) in 1st year to 8 (9%) in 4th year in 2013.	M 40% & test ML 60% & test	64	13%	Mastermaths

[2] For Institutions A, C and E, the BEd contains the standard 480 credits for the total programme. Institution B has 522 credits because it includes credits for compulsory courses that students from all faculties at Institution B are required to take. Institution D has a total of 510 credits for the BEd as the 30 credits awarded for teaching practice are not included in the standard 480 credits.

[3] These numbers reflect the total number of students from the designated areas reviewed (the areas in which the majority of the students at this university are located) who sat the exams in each of the mathematics modules in 2013. It is difficult to ascertain the total number of students in a cohort in the BEd IP programme at Institution C as there is flexibility in the length of time students may take to complete the degree. However, in 2013 1664 students from the designated areas sat the exam for the mathematics methodology course taken by all IP students and 1793 students sat the exam for the English methodology courses taken by all students. This suggests that a small proportion (around 10%) of Institution C's students are specialising in mathematics.

Four of the universities offer the specialist courses for mathematics to both intermediate phase and senior phase students together. Institution A is the exception, where the specialisation is offered for senior primary (Grades 4 to 7) students. This reflects the confusing reality in schools where, although there is a senior phase curriculum for Grades 7 to – 9), Grade 7 is mostly taught in primary schools and Grades 8 to 9 in high schools.

At Institution A and Institution B, IP maths students are given the option of doing the mathematics content courses designed for prospective high school mathematics teachers in place of those designed for prospective IP teachers. At each of these institutions a very small numbers of IP maths students (about 5) selected this option and thus it is not analysed in this report. At Institution C students are able to take courses in the Mathematics Department towards their BEd degree. In 2012 it was made a requirement for IP maths students to take these courses. However this resulted in very few students taking mathematics courses and thus the mathematics courses for IP were reintroduced into the Faculty of Education. As the majority of IP maths students will take the courses offered in the College of Education only these are discussed in this report.

The number of students specialising in intermediate phase mathematics is small at institutions A and E. At Institution A the cohort that graduated in 2013 was the first cohort to be offered specially-designed IP mathematics courses and the number of students taking these courses has not yet stabilised. At Institution E in 2013 there were 58 students in the first year course and eight students in the final year of the course. This differs from the pattern in 2012 when there were 24 students in the first year course and 16 students in the final year of the course. No general deduction can be made from the 2013 student number data at Institution E, except to say that the number of students specialising in IP mathematics tended to be relatively small. Although the absolute numbers specialising in mathematics at institutions C and D were larger, they represented less than one fifth of the IP students at each of those institutions.

At Institutions B and D, IP maths students are required to have passed mathematics in the National Senior Certificate (NSC) examinations. At Institutions A, C and E, those who did mathematical literacy for matric are able to specialise in mathematics. Both institutions A and E consider students' performance in the institutions' own tests for admission, which enables students with relatively low NSC maths marks (40% - 50%) to be accepted as IP maths students. Although lecturers' comments on their perceptions of the students' mathematical backgrounds varied, a theme that emerged from the interviews with lecturers (especially from Institutions A, B and E) was that the IP maths students tended to have a procedural approach to mathematics and considerable work was needed to build their conceptual foundations. A second theme that emerged was that the preparedness of students to study maths varied greatly and thus satisfying the needs of both those who were strong mathematically and those who were struggling was difficult. Although Institution C offers some tutorials for students, it was acknowledged that many students were unable to access these tutorials and the lack of support for students who encountered difficulties was regarded as a weakness. Institution B and E do offer support for struggling students. However, the lecturers at Institution E commented that although Mastermaths (a computer-based programme aligned to the South African curriculum) was available to students, very few made use of it.

At most of the universities the number of credits for the mathematics courses accounted for about one quarter of the total credits required for the BEd degree, making mathematics a substantial component of the degree. At Institution E, although the contact hours for the mathematics courses were approximately 500 hours, the total number of credits awarded for the courses was 64, accounting for only 13% of the credits for the degree.

2.2 Basic information about the mathematics offering for prospective intermediate phase teachers not specialising in mathematics

Table 2: Basic information about the mathematics offering for prospective IP teachers not specialising in mathematics

HEI	Approx. number of IP students not specialising in maths per year	Maths credits	Percentage of of total credits	Support offered
A	75	40	8%	Tutorials for the compulsory maths course
B	120	16	3%	Senior students
C	1600	12	2.5%	One optional tutorial per semester for some courses
D	312	68 (ML)	13%	
E	55-80	19	4%	Mastermaths, tutorials, peer tutoring

Institutions C and D again have large numbers of IP non-maths students; Institution B has roughly 120 students in each year; and Institutions A and E have smaller numbers of students. At Institution C, IP non-maths students are given 45-60 hours of mathematics methodology tuition and at Institution B IP non-maths students receive 40 hours of basic mathematics tuition with no methodology[4]. The nature of intermediate phase teaching means that many IP non-maths students will end up teaching mathematics at some point in their careers, but the very limited time these students spend on mathematics at institutions B and C is unlikely to prepare them adequately for that eventuality.

[4] Although this is currently the case, the interviewees at Institution B indicated that this will be changed when the current revision of the BEd is implemented.

At Institution D about 70 of the 200 hours of mathematics tuition offered to IP non-maths students were spent on methodology courses which focused on how to teach mathematical literacy in the FET phase. The decision to provide this tuition was justified on the basis that non-maths IP students will have taken mathematical literacy at school. However, if these students are going to be IP teachers, they will not teach the subject mathematical literacy. The approaches to teaching primary school learners mathematics and to teaching high school students mathematical literacy differ in significant ways. It is thus unlikely that the time spent on methodology for teaching mathematical literacy in the FET phase will be useful for the IP students.[5]

Lecturers from all the institutions commented that a large number of IP non-maths students had weak backgrounds in mathematics and struggled with the course.

3 Broad overview of the courses

The principal delivery mode, forms of assessment, teaching and learning materials and use of technology of each course described in Tables 1 and 2 are summarised in Table 3.

Table 3: Broad overview of the courses

HEI	Main mode of delivery	Assessment	Material used	Technology used
A	Lectures for large classes, interactive work in smaller classes	Mainly tests and exams for content, ½ exams, ½ assignments for methodology	Worksheets, readings and textbooks	Yes
B	Mixed – largely interactive	½ exams, ½ assignments	Comprehensive course packs with study notes and readings, textbook	Yes
C	Distance	Exam	Study guide and textbook	In a methodology course
D	Lectures	Mainly tests and exams	Notes, textbooks for some mathematics courses	Unclear

[5] Although this was the case in 2013, Institution D has indicated that the curriculum for 2014 has been changed so that IP non-maths students no longer do mathematical literacy content and method.

HEI	Main mode of delivery	Assessment	Material used	Technology used
E	½ formal lectures, ½ group work and self-study	Mainly tests and exams, some assignments	Own notes for mathematics, textbook for some methodology courses	In a methodology course

At most of the institutions interviewees indicated that the delivery mode for large classes was lectures, although they preferred interactive work and use it where possible. The mathematics courses were assessed largely through examinations and tests, whereas the methodology courses often included assignments and practical work. The material used and made available to the researchers varied. Institution B provided comprehensive course packs that are provided to students in advance and used in conjunction with prescribed textbooks. Institutions A and E seemed to issue notes, readings and worksheets to students as necessary and the material was less clearly packaged. It was not clear how complete the material provided by Institution D was and because, in many cases, no course outline was provided, it was hard to ascertain what material might have been made available to students.

In relation to the material used, it is particularly interesting to note that four of the institutions used a version of the textbook by Van de Walle, Karp and Bay-Williams: *Elementary and Middle School Mathematics*. Institution A used it in the methodology courses; Institution B used the book extensively for the methodology courses as well as some of the mathematics courses; Institution C used the book for both the mathematics and methodology courses; and Institution E used it in some of the methodology courses. This textbook originates in the USA and is used in a number of teacher education courses in that country.

Institutions A and B also used two other mathematics textbooks that were designed for prospective elementary school teachers in the USA. Although the textbooks[6] used are different, there is a large degree of similarity between the two books in terms of content and approach. This is true of many of the available textbooks for prospective elementary school mathematics teachers produced in the USA.

The ITERP study did not specifically collect information on the use of technology for teaching and learning mathematics, but it was clear from the data that both institutions A and B use technology in the teaching of the mathematics courses and expose students to mathematics software packages. Institutions B, C and E incorporate a component on technology in their methodology courses.

[6] Institution A uses *A Problem Solving Approach to Mathematics for Elementary School Teachers* by Billstein, Libeskind and Lott. Institution B uses *Mathematics for Elementary School Teachers* by O'Daffer, Charles, Cooney, Dossey and Schielack.

4 Analysis of the content of the courses

4.1 How the content of the courses was analysed

For each of the institutions, the courses taken by IP maths students and IP non-maths students were recorded separately. The material pertaining to each course, along with information from the interviews, was scrutinised and the course content analysed along the 8 dimensions discussed below. This information was then probed to pull out themes and important points which are discussed in sections 4.2.

The dimensions for analysis:

i) Course type:

This simply recorded whether the course was designated as a mathematics content course (Ma) or a mathematics methodology course (Me). Many methodology courses incorporate mathematics and many content courses incorporate methodology. The code given the course was on the basis of what the course was designated as in the curriculum outline and not on the basis of the actual content of the course.

ii) Mathematics content:

The particular topic areas in mathematics on which a course focused overtly were recorded (Table 4). For this purpose the categorisation used in the TEDS-M Study (Tatto, Schwille et al, 2008), which itself drew on the work of TIMSS 2007, was used, but adapted to suit the data.

Table 4: Mathematics content categories

Maths content	Content included
Number	Whole numbers Fractions and decimals Patterns and relationships Integers Ratios, proportions, and percentages Irrational numbers Number theory
Geometry	Geometric shapes Geometric measurement Location and movement

Maths content	Content included
Trigonometry	Trigonometry
Algebra and functions	Patterns Algebraic expressions Equations/formulas and functions
Calculus	Differential and integral calculus
Linear algebra	Linear algebra
Other higher mathematics	Topics apart from calculus and linear algebra that are done at university level
Data	Data organisation and representation Data reading and interpretation Chance
Financial Mathematics	Basic mathematics relating to financial situations Simple and compound interest Annuities

iii) The level of the mathematics content:

This dimension maps the mathematical content to where it would typically occur in the South African curriculum. This was designated as follows:

IP	SP	FET	Uni
The intermediate phase (grades 4 – 6). This would include the basic operations covered in the Foundations phase (grades 1 – 3)	The senior phase (grades 7 – 9)	The further education and training phase (grades 10 -12)	Mathematics typically taught in mathematics departments at University.

Note that the level of mathematical content does not indicate **depth**, but simply the stage in the curriculum where that content is typically encountered. For example, it is possible to pose questions on university level mathematics that simply require the application of well-rehearsed procedures (e.g. find the derivative of f(x) = 3 sin x) or to pose questions on IP or SP level mathematics that are more demanding (e.g. use a contextual situation to explain why when dividing a number by $\frac{2}{3}$, we multiply it by $\frac{3}{2}$).

iv) Education content

*W*here the course had a focus on issues related to teaching and learning the particular content was assessed. In doing this, categories for the education content of the courses were developed, working in a grounded way from the data. The categories are summarised in Table 5.

Table 5: Education content categories

Education content	Description
Theory	Theories of teaching and learning e.g. constructivism, problem-centred mathematics
Assess	Assessment
Curriculum	Issues related to the curriculum, learner teacher support material, drawing up year plans and lesson plans on the basis of curriculum content.
Metacog	Metacognition
Misconcep	A focus on misconceptions and the way to deal with misconceptions in mathematics learning and teaching.
Inclusivity	Issues relating to facilitating access for all learners to mathematics. This would include language issues in the mathematics classroom.
Tech	Technology in mathematics teaching and learning
Content	A focus on the teaching and learning of specific mathematics content e.g. van Hiele's stages in geometry learning, developing number sense
Strategy	Specific strategies for improving teaching and learning are put forward e.g. using lesson study or questioning and listening

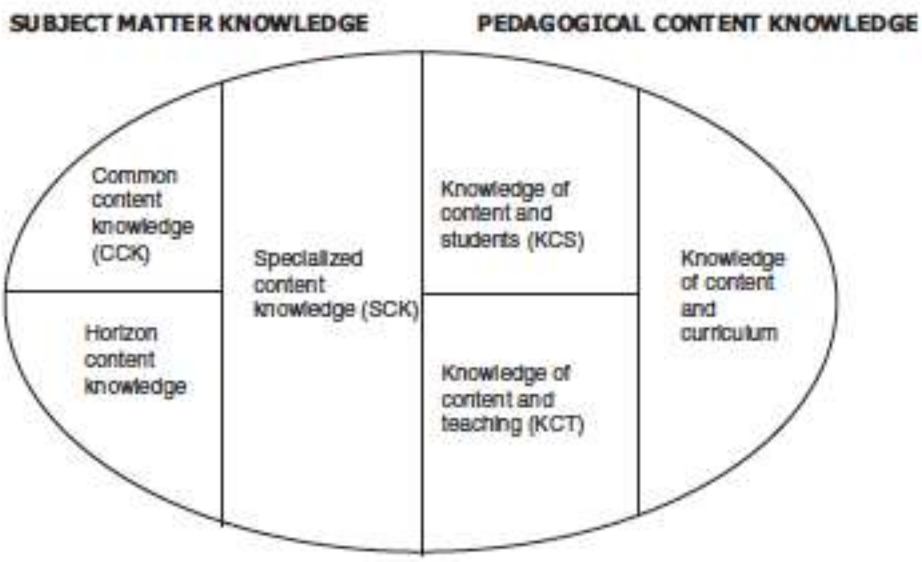
v) Number of lecture periods:

This captures the number of lecture periods or hours of contact time devoted to each piece of content or to the course as a whole, depending on the level of detail on contact time provided in the documentation received from each institution. In the case of Institution C, there are no contact hours. However, the institution recommends 15 weeks of 6–8 hours of self-study for each course. As most institutions assume approximately equal self-study to contact time, each Institution C course is estimated to be 3-4 hours per week of lectures.

vi) and vii) Subject matter knowledge and pedagogical content knowledge:

A core element drawn upon for the dimensions for analysis was the framework developed by Ball, Thames and Phelps (2008), which refines Shulman's work in this field and provides a useful way of thinking about the components of subject matter knowledge (SMK) and pedagogical content knowledge (PCK) in mathematics. The scheme is summarized in Figure 1.

Figure 1: Subject matter knowledge (SMK) and pedagogical content knowledge (PCK)



(Ball et al, 2008, p403)

In analysing the nature of the **subject matter knowledge** it was useful to distinguish between common content knowledge (CCK) and specialised content knowledge (SCK). CCK refers to mathematical knowledge and skills used by non- teachers, for example, knowing how to add two numbers, finding the maximum value of a function in context or understanding the connection between differentiation and rate of change are knowledge required by a variety of users of mathematics. SCK, on the other hand, is the mathematical knowledge and skills that are unique to teaching. Examples of SCK include being explicit about models for division as sharing and grouping, being able to create a contextual problem that would lead to the calculation $3 \div \frac{1}{2}$ or being able to analyse whether an atypical long multiplication algorithm is mathematically correct.

Horizon content knowledge refers to the relationship between mathematical topics across the curriculum. It was not a useful category for this analysis; rather the South African Curriculum categorisation of the mathematics content levels was more useful in this regard.

The subcategories within the field of PCK were not specified as these subcategories are broad, with substantial overlap and do not capture the content of the courses at the level that was considered necessary. Following the work of Parker (2008) and Rusznyak and Shalem (2013), and drawing on Muller's (2009) distinction between conceptual and contextual coherence, it was decided rather to look at the extent to which the pedagogical knowledge portrayed in the courses was related to concepts based in a body of educational knowledge and the extent to which the pedagogical knowledge was linked to practice. It also became apparent that it would be useful to look at the extent to which students needed to engage with mathematics itself in the exploration of the pedagogical issues.

Thus the pedagogical content knowledge of the courses was categorised using the symbols shown in Table 6.

Table 6: Categories describing the nature of pedagogical content knowledge

Category	Symbol	Strongly present ++	Present +	Absent -
Mathematics	M	Mathematical content is present and students are required to do the mathematics	Mathematical ideas are discussed or illustrated	Mathematics is only discussed in general, no specific mathematics is engaged with
Concept	C	Students engage with research papers and theoretical papers from the field of education research	Research from the field of education research is referenced but not engaged with	No research from the field of education is referenced
Practice	P	Students need to create an actual lesson plan, classroom activity or discuss observations from a lesson observed	Students discuss strategies for teaching or are given practical ideas for teaching	No real or imagined classroom situations evoked

viii) Level of cognitive demand:

In order to get a sense of the level of cognitive demand of the mathematical tasks in the courses the taxonomy shown in Table 7 was created. The taxonomy draws on the work of

Stein et al (2000) and the Learning Mathematics for Teaching Project (2011) on the Mathematical Quality of Instruction (MQI).

Table 7: Categories for characterising the level of cognitive demand

Category	Description
Knowledge (K)	Recall of facts, rules, formulae
Perform routine procedures (RP)	Perform procedures that have been seen previously
Make connections (C)	Between different representations
	Between mathematics and context
	With other mathematical topics
	Between the procedure and the underlying concept
Engage in mathematical practices (MP)	Investigate and generalise
	Provide explanations
	Justify and prove
	Solve non-routine problems

Each course at each of the institutions was then analysed according to the eight dimensions described above. These analyses are shown in detail in Appendices A and B. A summary of the salient points is presented below.

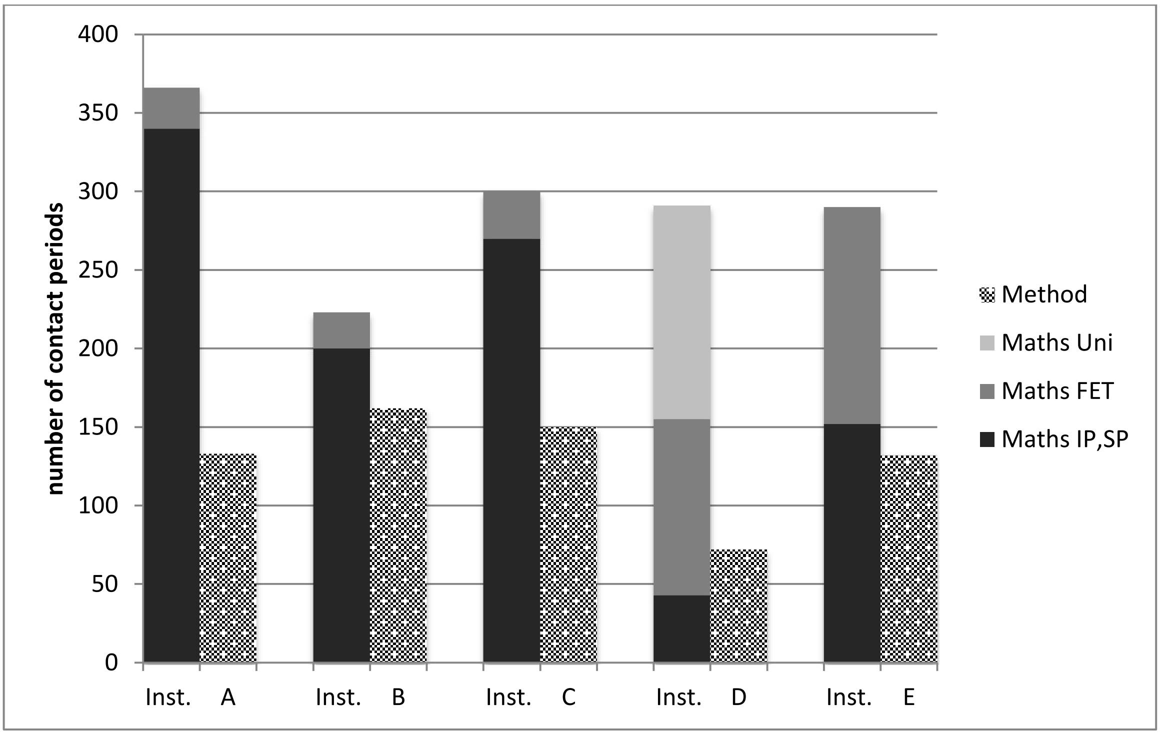
4.2 The scope and depth of the mathematics courses offered for prospective intermediate phase teachers

4.2.1 For those specialising in mathematics

The discussion in this section is based on the tables in Appendix A. It should be noted that Institutions A, B and C offer IP maths students the option of doing other mathematics courses (the courses taken by those intending to teach FET mathematics). However, the campus researchers indicated that few students take this option hence they have not been included in this analysis.

The graph below summarises the number of contact periods spent on mathematics courses and methodology courses at each of the institutions. It also provides a breakdown of the mathematics courses in terms of the level (IP, SP, FET or University) of the mathematics studied.

Figure 2: Number of contact periods and level of content for those specialising in mathematics[7]



From the graph it is apparent that all the universities spent more time on the mathematics content courses than on the mathematics methodology courses. At all the universities except Institution D the focus of the mathematics courses was on IP and SP level mathematics and thus ideas for teaching the content could be incorporated into class discussions in the mathematics classes. Notably, at Institution C the mathematics course and the methodology course made use of the same textbook, *Elementary and middle school mathematics* by van de Walle, Karp and Bay-Williams. This textbook incorporates mathematical tasks but has a strong focus on teaching methodology and ideas.

The focus of institution D's courses are at FET and University level, with a small amount of content at the SP level and a small number of hours for methodology. Thus students' engagement with the mathematics they will be teaching is limited.

Given that the mathematical level of the content courses at Institutions A, B, C and E was largely at the IP/SP level, it is important to assess the nature of that content. At Institutions A, B and C the vast majority of the mathematics courses tended to combine CCK and SCK. In addition the mathematics courses at these institutions tended to cover the full spectrum of cognitive demand, namely knowledge (K), routine procedures (RP), connections (C) and

[7]Institutions provided differing levels of detail about the weighting of the courses in terms of time. It was also unclear in some cases whether time referred to was hours or lecture periods (which might be slightly less than an hour). Thus in this graph we show contact periods (i.e. lecture or tutorial periods), but caution that there might be some differences between institutions in terms of how long those contact periods are. In particular, as Institution C is a distance institution, there are no contact hours per se. Each course at Institution C is stipulated as 15 weeks of 6–8 hours of self-study. As most institutions assume approximately equal self-study to contact time the "number of contact periods" shown for Institution C in the graph has been calculated as 3-4 hours per week.

mathematical practices (MP). Table 8 provides more detail on the knowledge types and cognitive tasks demanded by courses for maths students at the five institutions.

Table 8: Summary of the knowledge types and cognitive task covered by courses for maths students

Institution	Knowledge type (SCK, CCK)	Cognitive task (K, RP, C, MP)
A	Large majority of courses combine CCK and SCK	Full spectrum
B		
C		
D	Only CCK	K, RP
E	CCK, SCK	Very little MP

At Institution D the courses covered FET and University level content and contained CCK only. The majority of these courses focused on the level of recall of knowledge and performance of routine procedures. For example, in calculus the emphasis was placed on calculating derivatives and integrals. It thus seems like the higher-level content is tackled at a lower level of cognitive demand. It is questionable as to how useful this would be for enhancing the kind of knowledge required by IP Maths teachers.

In looking across all the mathematics courses at Institutions A, B, C and E where students revisited IP/SP mathematics, it was noticeable that developing tasks that push the students beyond simply re-doing school mathematics to revisiting it in a way that will deepen conceptual understanding and enhance their abilities as teachers (a goal that many of the lecturers espoused in lectures) required the development of high quality mathematics tasks which then needed to be trialled and improved.

Although each of these institutions had some strong aspects in their courses it seemed that a focus on working cooperatively to improve the quality of "IP/SP mathematics tasks for teachers" might be a fruitful avenue for the institutions to pursue. Examples of mathematics tasks which elicit a range of cognitive skills follow.

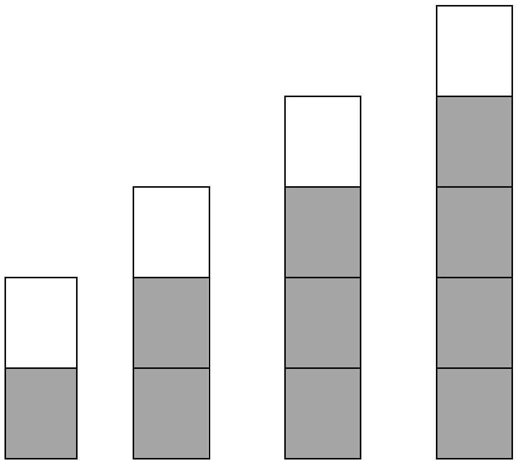
Examples of tasks drawing on specialised content knowledge

Example 1

Draw a sketch to illustrate 35% of 270 kilometres (Institution B, MALA221 exam)

Example 2

You see the following poster for sale. Would you buy it for use in your maths classroom? Give reasons for your answer.



$$\frac{1}{2} < \frac{2}{3} < \frac{3}{4} < \frac{4}{5}$$

(Institution A, EDUC2195, question from class worksheet)

Examples of tasks at the level of cognitive demand of connections (C) or mathematical practices (MP)

Example 3

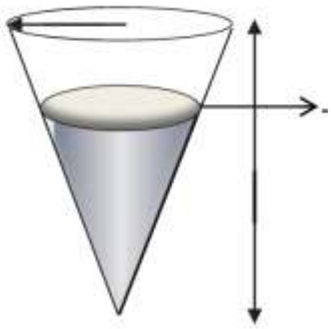
To celebrate World Maths Day a school asks each of the learners from the senior grades to partner with a learner from the junior grades so they can work together as a team on a maths quiz on World Maths Day. So far 2/3 of the learners from the senior grades have partnered with 3/5 of the learners from the junior grades. What proportion of learners at the school have got partners for World Maths Day?

(Institution A, EDUC2195, question from class worksheet)

Example 4

A cone is filled with water up to the three-quarter mark as shown in the diagram. Think about the situation. Do you think that the volume of the water is a three quarter of the volume of the cone? Write down what you think.

Now do the necessary calculations to find the ratio of the volume of water in the cone: volume of the cone



(Institution C, MAE103L, question from assignment)

At Institution E, where much of the mathematics is pitched at FET level, although courses contained some SCK, there was a greater weighting of CCK. Example 5 illustrates the point.

Example 5

In the test for the 3rd year mathematics course, students were asked to simplify a number of complicated expressions involving surds and exponents

e.g. $\left(\frac{\sqrt{8}-\sqrt{72}}{\sqrt{32}-\sqrt{18}}\right)^{-2}$

(Institution E, MMS301S)

In addition, the material contained very few instances of the MP level of cognitive demand and most of the connections were to 'real life' contexts and less to conceptual underpinnings, as shown in the following example.

Example 6

Quick Plumbing company charges R60 to be called out for repair work and then R20 for each 30 minutes of labour. *Fast Plumbing* charges R40 call-out fee and R30 for each 30 minutes of labour.

a) Determine the equation to calculate the costs for each of the companies. Tables may be used.

b) Plot the graphs on the same set of axes. Use the grid at the back of your paper.

c) When will the cost for each company be the same? Find this through calculation.

d) Which company would you hire for a 5-hour job? Say why.

Such examples were then followed by work on a number of other questions involving algebraic manipulation or graphical functions. These tasks are not easy, although they are questions that would be expected of a learner in Grades 10 – 12 to tackle in mathematics. However, They are different in nature to the specialised content knowledge questions and to the questions requiring deep connections between mathematical concepts or the use of problem-solving techniques illustrated above.

In terms of methodology, there was considerable variation across the institutions in terms of how this was handled, as shown in Table 9.

Table 9: Nature of pedagogical content knowledge in courses for maths students

HEI	Links between knowledge components and practice (C, P, M)
A B	C++, P++, M++ Conceptual underpinning linked to practice and a strong mathematical gaze. Students read maths education research, engage with math tasks and use their knowledge of both to explore options for practice
C	C-, P+, M++ Strong math focus; moderate to strong link to practice; conceptual underpinnings of the field not foregrounded
D	C-, P++, M+ Conceptual underpinnings marginalised; strong focus on practice, with a particular emphasis on the SA curriculum
E	No consistent pattern: a mix of all categories

At Institutions A and B the C++, P++, M++ pattern predominated in the methodology courses, indicating a linking of the conceptual underpinning with practice and a strong mathematical gaze. At both Institutions A and B students read mathematics education research articles, engaged with mathematical tasks and were asked to use their knowledge of both of these to explore options for practice.

IP maths students in their final year at Institution B engage in a lesson study and write up the results of this as part of their mathematics methodology course. Lesson study would encourage a strong coherence between the mathematical, conceptual and practical elements as the student teachers would need to use mathematical knowledge and consult existing research to design a lesson and then reflect on and improve this lesson after implementing it in practice.

At Institution C, the use of the van de Walle textbook as the main source for both the mathematics and methodology courses meant there was a strong mathematical focus, with a moderate to strong link to practice. However the conceptual underpinnings of the field, although referenced by van de Walle et al. were not foregrounded.

At Institution D the conceptual underpinnings appeared to have been marginalised and a strong focus was placed on practice – with a particular emphasis on getting to grips with the South African curriculum.

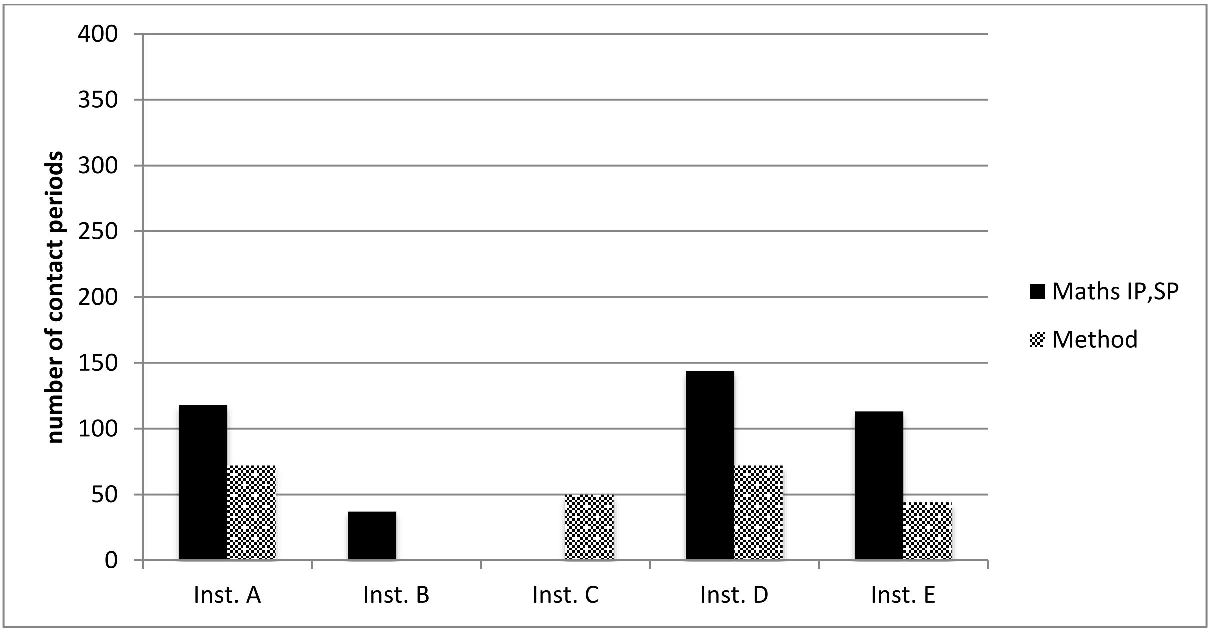
At Institution E there was a less clear pattern in terms of the mathematical, practice based and conceptual underpinnings. Nevertheless, IP maths students in their final year at Institution E are expected to do a research project as part of their mathematics methodology course. This kind of research project has the potential to allow students to bring together the conceptual and practical elements of mathematics education in a powerful way.

4.2.2 For those not specialising in mathematics

The discussion in this section is based on the tables in Appendix B. It should be noted that Institutions A and E do offer IP non-maths students the option of taking an additional elective course. These have not been included in the following summary but are included in Appendix B.

The graph below summarises the number of hours spent on mathematics courses and methodology course at each of the institutions[8].

Figure 3: Number of contact periods for those not specialising in mathematics



The most striking observation is that at Institution B, IP non-maths students do not do a methodology course[9]. The mathematics offered to these students consists of courses that all

[8] Institutions provided differing levels of detail about the weighting of the courses in terms of time. It was also unclear in some cases whether time referred to was hours or lecture periods (which might be slightly less than an hour). Thus in this graph we show contact periods (i.e. lecture or tutorial periods), but caution that there might be some differences between institutions in terms of how long those contact periods are. In particular, as Institution C is a distance institution, there are no contact hours per se. Each course at Institution C was stipulated as 15 weeks of 6–8 hours of self-study. As most institutions assume approximately equal self-study to contact time the "number of contact periods" shown for Institution C in the graph has been calculated as 3-4 hours per week.

BEd students do and are “mathematics for everyday life” and thus do not contain SCK. At Institution C, IP non-maths students only do a methodology course. However, this course does have a strong mathematical focus.

Institutions E, A and D offer both mathematics and methodology courses to the IP non-maths students. However at Institution D these courses are mathematical literacy courses. The mathematical content covered in the mathematical literacy course is aligned with IP and SP mathematics. However, the methodology courses look specifically at the subject mathematical literacy, which is only offered at FET level. As noted earlier the approach to teaching mathematical literacy as an FET subject (which is the focus of these methodology courses) differs substantially from the approach required to teach mathematics as an IP subject. Thus it is unlikely that the Institution D methodology courses for IP non-maths students prepares the students for the classroom situations they are likely to face[10].

At Institution E the non-mathematics IP students cover the same content as the IP maths students during the first two years. However, there is a greater focus on tasks at the lower level of cognitive demand (knowledge and routine procedures) than in the courses for the IP maths students and they are taught and examined separately. Because the non-mathematics IP students’ mathematics course focuses largely on tasks at the level of knowledge or performing routine procedures, the course tends to focus almost entirely on CCK. At institution E the methodology courses taken by these students are the same, but the non-maths IP students only do the first two years of the methodology course.

At Institution A the non-maths IP students do a course taken by all BEd students. This course covers the key areas of IP and SP mathematics and incorporates both CCK and SCK. Although the course deals with questions at all levels of cognitive demand, the assessment tends to focus on lower level of cognitive demand. The methodology course is the same as that taken by the IP maths students.

5 Concluding comments

This report indicates that there are strong commonalities in the mathematics content courses in four of the institutions studied (Institutions A, B, C and E). These commonalities align with the codified version of “mathematics for elementary school teachers” exemplified in the textbooks of the same name from the USA. The mathematical work in these courses focuses mainly on the mathematics content that South African learners will deal with in the IP and SP (i.e. Grades 4 – 9), but in most cases is dealt with at a much deeper level than expected at school and with a specific focus on the specialised content knowledge required by teachers. Institution D, on the other hand, focuses on mathematical content typically taught to university students. As no specific attempt is made to link this knowledge to teaching it is unclear how much this will translate to the classroom.

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[9] Although this is the case for the BEd programme in existence in 2013, Institution B indicated that they intend to change this in their revised curriculum.

[10] As noted earlier, Institution D has indicated that this was changed in 2014, but the new 2014 offerings have not been examined by this project.

There is far more diversity in the mathematics methodology courses offered across the institutions and is thus very difficult to draw out emerging themes.

Similarly there is great variability in the offerings to prospective IP teachers not specialising in mathematics. Given that many IP teachers will end up teaching mathematics at some level, even if they have not specialised in maths, there is some cause for concern about the lack of preparation they will receive in their pre-service education emerging from the findings of this research. This is an area that needs considerable attention.

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Appendix A: Analysis of the courses for IP Mathematics specialists

Table 10: Institution A Mathematics Specialisation

Institution A Mathematics Specialisation												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematical Routes	Y	Y	Geometry	IP/SP	Content	43	Ma	CCK SCK	P+ M++ C-	K, RP, C@, MP@	@In the course material, but only minimally in the assessment.
		Y	Y	Number	IP/SP	Content	47	Ma		P++ M++ C-	K, RP, C@, MP@	
		Y	Y	Algebra	SP	Content	3	Ma	CCK	P+ M++ C-	K, RP, C@, MP@	
		Y	Y	Data	IP/SP	Content	25	Ma	CCK SMK*	P+ M++ C-	K, RP, C@	* A small amount

2	Concepts in Literacy in Mathematics I	Y	Y	Number	IP/SP	Content	84	Ma	CCK SMK	P+ M++ C-	K, RP, C, MP	
2	Senior Primary Methodology: Mathematics I	Y	Y	Number	IP/SP	Content	72	Me	SMK	P++ M++ C++		Strong mathematical focus in the course
		Y	Y			Theory		Me		C++ P++ M+		
3	Concepts and Literacy in Mathematics II	Y	Y	Geometry	IP/SP/FET		56	Ma	CCK[%]		K, RP, C, MP	[%]Although largely CCK, it does focus on a deep conceptual understanding
		Y	Y	Data	IP/SP/FET		24	Ma	CCK SMK[@]		K, RP, C[@]	
4	Concepts and Literacy in Mathematics III	Y	Y	Algebra and functions	IP/SP/FET		84	Ma	CCK SMK[@]		K, RP, C, MP	

	Senior Primary Methodology: Mathematics II	Y	Y	Geometry	IP/SP	Content	16	Me		M++ P++ C++	K, RP, C, MP	
		Y	Y			Theory Inclusivity	24	Me		C++ P++ M+		
		Y	Y	Algebra and functions		Content	12	Me	CCK	C+ M++ P++	K, RP, C, MP	
		Y	Y			Assess	9	Me		C++ P++ M+		

Table 11: Institution B Mathematics Specialisation

Institution B Campus A - Mathematics Specialisation												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
2	Maths FET 2	Y	Y	Number	IP, SP		8	Ma	CCK		K, RP, C	
2		Y	Y	Data	IP, SP		7	Ma	CCK		K, RP, C	
2		Y	Y	Financial maths	SP, FET		4	Ma	CCK		K, RP, C	
2	Maths Content IP/SP 2.1	N	N	Number	IP, SP	Content Metacog	48	Ma	CCK, SCK	C++ P+ M+	C, MP	No course material/exam available. Used interview
2	Maths Content IP/SP 2.2	Y	Y	Number	IP, SP	Content Metacog	48	Ma	CCK, SCK	C+ P+ M++	K, RP, C, MP	
2	MATD211:	Y	Y	Senior phase mathematics	SP	Theory Assess Curric	36	Me		C++ P++ M++	Teach	Exam incorporates C++ and M++. Also course has P++ assignments

3	Maths Content FET 3	N	N	Geometry	IP		18	O	CCK			No course material or exam available. Used interview
3	Maths Content IP/SP 3.1	N	Y	Data	IP, SP, FET		48 for course	Ma	CCK, SCK		K, RP, C, MP	No course material available Used interview and exam
3		N	Y	Algebra and Functions	SP	Content		Ma	CCK, SCK	M+ P+ C?	K, RP, C	
3		N	Y			Metacog		Ma		M++ C++ P-		
3	Maths Content IP/SP 3.2	Y	Y	Geometry	IP, SP, FET	Content	42	Ma	CCK, SCK	C++ P+ M+	K, RP, C, MP	
3		Y	Y			Metacog	6	Ma		C++ P- M+		

3	MATD312	Y	Y	Number	IP	Content Miscon Assess	24	Me	SCK	C++ P+ M++	Teach	
4	MATD413	Y	Y			Strategies, Metacog	48	Me		C++ P++ M+		
4	MATD421	y	y	Algebra and Functions Geometry		Content	48	Me		C++ P++ M+	Teach	
4		Y	Y			Curric Inclusivity Theory Tech		Me		C++ P++ M+		

Table 12: Institution C Mathematics Specialisation

Institution C Mathematics Specialist												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	**Comments**
1	Maths Content Numbers and Operations	Y	N	Number	IP, SP	Content	15 weeks max	Ma	CCK SCK	M++ P+ C+	K, RP, C, MP	Van de Walle textbook used in the MAE courses and in PST course
1	Maths Content Space, Geometry and Trigonometry	Y	N	Geometry	IP, SP, FET	Content	15 weeks max	Ma	CCK (SCK)	M++ P+ C+	K, RP, C, MP	(SCK not so directly in the maths – more van Hiele discussion around maths)
1	Maths Content Measurement	Y	N	Geometry	IP, SP	Content Theory	15 weeks max	Ma	CCK SCK	M++ P+ C+	K, RP, C, MP	

1	Assessment in IP Maths	Y	Y			Assessment	15 weeks max	Me		M+ C+ P++		The focus of the exam is on practice with minimal C.
2	Mathematics Education Method	Y	N			Theory	15 weeks max	Me		C+ M+ P+		
		Y	N	Geometry		Content				M++ C+ P+		
		Y	N	Number		Content				M++ C+ P+		
		Y	N			Assessment				M+ C+ P+		

2	Algebra	Y	Y	Algebra and functions	IP, SP, FET	Content	15 weeks max	Ma	SCK CCK	M++ P++ C+	K, RP, C, MP	
2	Statistics	Y	N	Data Handling	IP, SP, FET	Content	15 weeks max	Ma	CCK (SCK)	M++ P++ C+	K, RP, C, MP	
2	Financial Maths	Y	Y	Financial Maths	IP, SP	Content	15 weeks max	Ma	CCK	M++ P+ C-	K, RP, C	C is because connected to everyday life Pedagogy not in final exam
2	Maths – Tech and Media	Y	Y			Technology	15 weeks max	Me		M++ P++ C+		

Table 13: Institution D Mathematics Specialisation

Institution D Mathematics Specialisation												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematics 1A	N	Y	Algebra and Functions	FET		39	Ma	CCK		RP, C*	*Just some applied problems
		N	Y	Calculus	FET, Uni				CCK		RP, C*	
		N	Y	Linear Algebra	Uni				CCK		RP, C*	
		N	Y	Higher maths	Uni				CCK		RP, C*	
1	Mathematics 1B	N	Y	Algebra and functions	FET, Uni		9	Ma	CCK		K, RP, C*	
		N	Y	Geometry	SP, FET		18		CCK		K, RP	
		N	Y	Trigonometry	FET		9		CCK		K, RP, C*	
2	Mathematics 2A	N	Y	Higher Maths	Uni		36	Ma	CCK		K, RP	

		N	Y	Number	SP, FET				CCK		K, RP	
2	Mathematics 2B	N	Y	Calculus	Uni		12	Ma	CCK		K, RP	Seems similar to 01A
		N	Y	Linear Algebra	Uni		21		CCK		K, RP, C*	Seems similar to 01A
3	Mathematics 3A	Y	Y			Assess	36	Me		C+ P++ M+		
		N	Y			Curric		Me		P++ M/C?		
		N	Y			Content – number		Me		P+ M+ C?		
		N	Y			Content – algebra		Me		P+ M+ C?		

		N	Y			Metacog		Me		M++ C+ P++		
3	Mathematics 3B	N	N				36	Me				No info available. Interview seems to suggest assessment as focus
4	Mathematics 4A	N	N	Statistics and probability	FET		36	Ma	CCK			No info available
4	Mathematics 4B	N	Y	Financial Maths	FET		36	Ma	CCK		K, RP, C*	
4	Mathematics 4A	N	N	(Mensuration)			36	Ma				Mensuration stated in handbook
4	Mathematics 4B	N	Y	Calculus	Uni		36	Ma			K, RP	States transformation geom in handbook

Table 14: Institution E Mathematics specialisation

Mathematics specialisation at Institution E												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematics	Y	Y	Number	IP		12	Ma	CCK SCK*		K, RP, C*	*in course only, not in final exam
		Y	Y	Geometry	IP		22	Ma	CCK		K, RP	
		Y	Y	Algebra and Functions	IP, SP		10	Ma	CCK		K, RP, C*	*in course only, not in final exam
1	Curriculum Studies	Y	Y			Inclusivity	2	Me		M- C+ P+		
		Y	Y			Theory	4	Me		M+ C+ P+		
		N	N	Number		Content	2	Me				No course material so PCK hard to judge

		N	N	Algebra		Content	3	Me					No course material available
		Y	Y	Geometry		Content	9	Me		M+ C+ P+			
						Curriculu m	4	Me		P++			No course material so can't judge C. P++ as they design lesson plans and study curric
2	Mathematic s	Y	Y	Data handling	IP, SP	curriculu m	18	Ma	CCK		K, RP, C		Content is CCK. Curric added at end. C is because some connections to real life examples made
		Y	Y	Number	IP, SP	content	27	Ma	CCK		K, RP, C*		*in course only, and only minimally n final exam
		Y	Y	Geometry	IP, SP	(Content, Theory)	12	Ma	CCK		K, RP,		Van Hiele mentioned in outline, but no notes seen. Not tested.
		Y	Y	Algebra and functions	SP		9	Ma	CCK		K, RP		

2	Curriculum Studies	Y	Y	Number	IP, SP	Content	7	Me	SCK	C+ P++ M++		Design a fractions lesson
		N	Y	Data handling	IP	Content	2	Me		C+ P+ M+		No course material available so harder to judge. Used exam
		N	N	Geom		Content	4	Me	SCK	C+ P+ M++		No course material available so harder to judge. Used exam
		N	N			Strategy	3	Me		C+ ?		Listening, questioning, mind maps mentioned in outline. No material
		N	N			Theory	2	Me		C+? ?		Piaget, Mathematical proficiency, mentioned in outline. No material
		N	N			Curriculum	2	Me		P++ ?		Lesson planning mentioned in outline. No material

3	Mathematics	Y	Y	Algebra and functions	SP/FET		28		CCK		K, RP, C*	Outcomes stated in terms of NCS LO, AS for FET *in course only, and only minimally n final exam
		Y	Y	Geometry and measurement	SP/FET		36		CCK		K, RP, C, MP*	Some of notes are from gr 11 textbook *proof
		Y	Y	Data handling	SP/FET		24		CCK		K, RP, C	Notes state “FET Prob”
3	Curriculum Studies	Y	Y	Functions and algebra		Content	10		SCK		C+ P++ M++	
		Y	Y			Assess	4			C+ P+ M+		
		N	N	Geometry		Content	14					No material available
		Y	Y	Data handling		Content	8			C+ P? M++		

						Theories	4			C++ P+ M+		
						Tech	4			P++ C- M -		
						Curriculum	2			P++		No material , but states lesson plans
4	Mathematics	Y	Y	Functions and algebra	FET		32		CCK		K, RP, C	
		N	Y	Trigonometry	FET		16		CCK		K, RP, C	
		N	N	Geometry	FET		24		CCK			No material available
		N	Y	Financial Maths	SP		4		CCK		C	
		N	N	Calculus	FET		16		CCK			No material available
4	Curriculum Studies	Y	Y			Research	30				C++ P++	
		N	Y	Geometry		Content	2					No material available

		Y	Y	Mathematical literacy		Curriculum	6				C- P+ M-	
			Y			Inclusivity	4				C? P++ M-	Barriers to learning

Appendix B: Analysis of the courses for IP students who will not be mathematics specialists

Table 15: Institution A Generalist

Institution A generalist												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematical Routes	Y	Y	Geometry	IP/SP	Content	43	Ma	CCK SCK	P+ M++ C-	K, RP, C@, MP@	@In the course material, but only minimally in the assessment.
		Y	Y	Number	IP/SP	Content	47	Ma		P++ M++ C-	K, RP, C@, MP@	
		Y	Y	Algebra	SP	Content	3	Ma	CCK	P+ M++ C-	K, RP, C@, MP@	
		Y	Y	Data	IP/SP	Content	25	Ma	CCK SMK*	P+ M++ C-	K, RP, C@	* A small amount

2	Senior Primary Methodology: Mathematics 1	Y	Y	Number	IP/SP	Content	72	Me	SMK	P++ M++ C++		Strong mathematical focus in the course
		Y	Y			Theory		Me		C++ P++ M+		
	Senior Primary Methodology : Mathematics II Elective	Y	Y	Geometry	IP/SP	Content	16	Me		M++ P++ C++	K, RP, C, MP	
		Y	Y			Theory Inclusivity	24	Me		C++ P++ M+		
		Y	Y	Algebra and functions		Content	12	Me	CCK	C+ M++ P++	K, RP, C, MP	

		Y	Y			Assess	9	Me		C++ P++ M+		

Table 16: Institution B Generalist

Institution B (campus A) Mathematics Generalist													
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**		
2	Mathematical numeracy/ literacy (compulsory course) (2)	Y	Y	Number	IP/SP		8	O	CCK		K, RP, C		
		Y	Y	Data and stats	IP/SP		7	O	CCK		K, RP, C	Tech	
		Y	Y	Financial maths	SP/FET		4	O	CCK		K, RP, C	Tech	
3	*Mathematical numeracy/ literacy (compulsory course) (3)*	*N*	*N*	*Geometry*	*IP*		*18*	*O*	*CCK*		*?*		*No course material or exam available. Used interview*

Table 17: Institution C Generalist

Institution C generalist												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
2	Mathematics Education Method	Y	N			Theory		Me		C+ M+ P+		Use van de Walle for this course
				Geometry		Content				M++ C+ P+		
				Number		Content				M++ C+ P+		
						Assessment				M+ C+ P+		

Table 18: Institution D Generalist

Institution D generalist												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematics 1A	N	Y	Number	IP/SP		36	Ma	CCK		K, RP, C*	*Just some very basic applied problems
		N	Y	Algebra and functions	IP/SP			Ma	CCK		K, RP, C*	
		N		Financial Maths	IP/SP							No material seen
1	Mathematics 1B	N	Y	Geometry	IP/SP		36	Ma	CCK		K, RP, C*	
		N	N	Trigonometry	FET							No material seen
2	Mathematics 2A						36	Ma				No material seen
2	Mathematics 2B						36	Ma				No material seen
3	Mathematics 3A	Y				Theory	36	Me		C+ P- M-		On ML and approaches to teaching ML

						Curric				C- P+ M-		
						Assess				C- P+		
3	Mathematics 3B	Y	Y			Theory	36	Me		C+ M- P+		
		Y	N			Curric		Me		P++ C+ M+		

Table 19: Institution E Generalist

Institution E generalist												
YOS	**Course code**	**Course notes seen**	**Exam seen**	**Maths content**	**Level**	**Education content**	**No. lectures**	**Course type**	**SMK**	**PCK**	**Cognitive demand of maths**	
1	Mathematics Education 1	Y	Y	Number	IP		12	Ma	CCK SCK*		K, RP, C*	*in course only, not in final exam
		Y	Y	Geometry	IP		22	Ma	CCK		K, RP	
		Y	Y	Algebra and Functions	IP/SP		10	Ma	CCK		K, RP, C*	*in course only, not in final exam
1	Curriculum Studies	Y	Y			Inclusivity	2	Me		M- C+ P+		
		Y	Y			Theory	4	Me		M+ C+ P+		
		N	N	Number		Content	2	Me				No course material so PCK hard to judge

		N	N	Algebra		Content	3	Me				No course material available
		Y	Y	Geometry		Content	9	Me		M+ C+ P+		
						Curriculum	4	Me		P++		No course material so can't judge C. P++ as they design lesson plans and study curric
2	Mathematics Education II		Y	Data handling	IP/SP	curriculum	18	Ma	CCK		K, RP, C	Content is CCK. Curric added at end. C is because some connections to real life examples made
			Y	Number	IP/SP	content	30	Ma	CCK		K, RP, C*	* minimally n final exam
			Y	Geometry	IP/SP	(Content, Theory)	12	Ma	CCK		K, RP,	Van Hiele mentioned in outline, but no notes seen. Not tested.
			Y	Algebra and functions	SP		9	Ma	CCK		K, RP	

2	Curriculum Studies II	Y	Y	Number	IP/SP	Content	7	Me	SCK	C+ P++ M++		Design a fractions lesson
		N	Y	Data handling	IP	Content	2	Me		C+ P+ M+		No course material available so harder to judge. Used exam
		N	N	Geom		Content	4	Me	SCK	C+ P+ M++		No course material available so harder to judge. Used exam
		N	N			Strategy	3	Me		C+ ?		Listening, questioning, mind maps mentioned in outline. No material
		N	N			Theory	2	Me		C+? ?		Piaget, Mathematical proficiency, mentioned in outline. No material
		N	N			Curriculum	2	Me		P++ ?		Lesson planning mentioned in outline. No material

3	Mathematics Education III Elective	Y	Y	Algebra and functions	SP/FET		28		CCK		K, RP	
		N	Y	Geometry	IP/SP		36		CCK		K, RP, C	
		N	Y	Data handling	FET		24		CCK		K, RP, C*	*minimal